NAME _____ DATE _____ PERIOD _____

4-1 Study Guide and Intervention

The Distributive Property

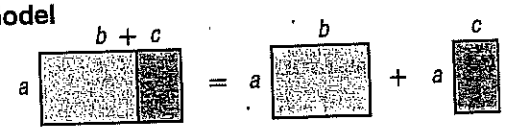
Numerical Expressions The expressions $2(1 + 5)$ and $2 \cdot 1 + 2 \cdot 5$ are equivalent expressions because they have the same value, 12. The **Distributive Property** combines addition and multiplication.

Symbols

$a(b + c) = ab + ac$

$(b + c)a = ab + ac$

Model



The Distributive Property also combines subtraction and multiplication.

Symbols

$a(b - c) = ab - ac$

$(b - c)a = ab - ac$

Example 1 **Use the Distributive Property to write $2(6 + 3)$ as an equivalent expression. Then evaluate the expression.**

$2(6 + 3) = 2 \cdot 6 + 2 \cdot 3$

$= 12 + 6$ Multiply.

$= 18$ Add.

Example 2 **Use the Distributive Property to write $5(9 - 3)$ as an equivalent expression. Then evaluate the expression.**

$5(9 - 3) = 5 \cdot 9 - 5 \cdot 3$

$= 45 - 15$ Multiply.

$= 30$ Subtract.

Exercises

Use the Distributive Property to write each expression as an equivalent expression. Then evaluate the expression.

1. $3(8 + 2)$

2. $2(9 + 11)$

3. $5(19 - 6)$

4. $-6(3 + 14)$

5. $(17 - 4)3$

6. $(5 + 3)7$

7. $9(20 + 8)$

8. $(8 - 3)4$

9. $7(40 - 5)$

Lesson 4-1

NAME _____ DATE _____ PERIOD _____

4-1 Study Guide and Intervention *(continued)*

The Distributive Property

Algebraic Expressions The Distributive Property can also be used with algebraic expressions containing variables.

Example 1 **Use the Distributive Property to write $7(m + 5)$ as an equivalent algebraic expression.**

$7(m + 5) = 7m + 7 \cdot 5$

$= 7m + 35$ Simplify.

Example 2 **Use the Distributive Property to write $3(n - 8)$ as an equivalent algebraic expression.**

$3(n - 8) = 3[n + (-8)]$ Rewrite $n - 8$ as $n + (-8)$.

$= 3n + 3 \cdot (-8)$ Distributive Property

$= 3n + (-24)$ Simplify.

$= 3n - 24$ Definition of subtraction

Exercises

Use the Distributive Property to write each expression as an equivalent expression.

1. $3(d + 4)$

2. $(w - 5)4$

3. $-2(c + 7)$

4. $9(b + 4)$

5. $(p - 10)8$

6. $-11(g - 6)$

7. $-14(j + 3)$

8. $(15 - a)20$

9. $9(50 + h)$

10. $5(12 - c)$

11. $-12(s - 2)$

12. $8(x + 60)$

13. $(y - 13)20$

14. $-15(4 + n)$

15. $7(r - 11)$

NAME _____ DATE _____ PERIOD _____

4-1 Word Problem Practice

The Distributive Property

1. **FARMING** Mr. Johannsen has a farm with 3 cows, 8 chickens, and some ducks. If the total number of farm animal legs is 40, how many ducks does Mr. Johannsen have on his farm?

2. **STAMPS** Amy buys retired stamps from the U. S. Postal Service catalog. Last month, she bought 8 $0.37 Candy Hearts, 8 $0.48 Niagara Falls, and 8 $0.80 Special Olympics stamps. How much did Amy spend on stamps in all?

3. **FUND-RAISING** The table shows the cookie sales for Tina's troop. If each box costs $3.50, show two ways that Tina could find the troop's total cookie sales.

Kind of Cookie	Number of Boxes
Mint	60 boxes
Vanilla sandwich	42 boxes
Peanut butter	56 boxes

4. **GEOMETRY** Jonah drew two squares with the same dimensions. He then added 2 inches to the length of one square to make it a rectangle. He also added 2 inches to the width of the other square to make it a rectangle. Write an equation that compares the perimeters of the two rectangles.

5. **SAVINGS** Daniel wants to buy a bicycle that costs $200.00. He saves the same amount each month from the money he earns mowing lawns. He also saves $15.00 of his monthly allowance.

   a. If x represents the amount he earns mowing lawns each month, write an expression to show Daniel's total savings after 8 months.

   b. If Daniel earns $25 each month mowing lawns, how long will it take him to save enough money to buy his bicycle?

Lesson 4-1

NAME _____ DATE _____ PERIOD _____

4-1 **Enrichment**

What Day Was It?

To find the day of the week on which a date occurred, follow these steps.

- Use the formula $s = d + 2m + \left[\frac{3(m+1)}{5}\right] + y + \left[\frac{y}{4}\right] - \left[\frac{y}{100}\right] + \left[\frac{y}{400}\right] + 2$

  where s = sum,
  d = day of the month, using whole numbers from 1 to 31,
  m= month, where March = 3, April = 4, and so on, up to December = 12; then January = 13 and February = 14, and
  y = year except for dates in January or February when the previous year is used.

- Evaluate expressions inside the special brackets [ ] by dividing, then discarding the remainder and using only the whole number part of the quotient.
- After finding the value of s, divide s by 7 and note the remainder.
- The remainder 0 represents Saturday, 1 represents Sunday, 2 represents Monday, and so on to 6 represents Friday.

Example **On December 7, 1941, Pearl Harbor was bombed. What day of the week was that?**

Let $d = 7$, $m = 12$, and $y = 1941$.

$$s = d + 2m + \left[\frac{3(m+1)}{5}\right] + y + \left[\frac{y}{4}\right] - \left[\frac{y}{100}\right] + \left[\frac{y}{400}\right] + 2$$

$$s = 7 + 2(12) + \left[\frac{3(12+1)}{5}\right] + 1941 + \left[\frac{1941}{4}\right] - \left[\frac{1941}{100}\right] + \left[\frac{1941}{400}\right] + 2$$

$$s = 7 + 24 + \left[\frac{39}{5}\right] + 1941 + \left[\frac{1941}{4}\right] - \left[\frac{1941}{100}\right] + \left[\frac{1941}{400}\right] + 2$$

$$s = 7 + 24 + 7 + 1941 + 485 - 19 + 4 + 2$$

$$s = 2451$$

Now divide s by 7. $2451 \div 7 = 305$ R1

Since the remainder is 1, December 7, 1941, was a Sunday.

Exercises **Use the formula to solve each problem.**

1. Verify today's date.
2. What will be the day of the week for April 13, 2012?
3. On what day of the week was the signing of the Declaration of Independence, July 4, 1776?
4. On what day of the week were you born?

NAME _____ DATE _____ PERIOD _____

4-5 Study Guide and Intervention

Solving Two-Step Equations

Solve Two-Step Equations A two-step equation contains two operations. To solve two-step equations, use inverse operations to undo each operation in reverse order. First, undo addition/subtraction. Then, undo multiplication/division.

Example 1 **Solve $\frac{c}{2} - 13 = 7$. Check your solution.**

$\frac{c}{2} - 13 = 7$	Write the equation.	**CHECK:** $\frac{c}{2} - 13 = 7$
$\frac{c}{2} - 13 + 13 = 7 + 13$	Addition Property of Equality	$\frac{40}{2} - 13 \stackrel{?}{=} 7$
$\frac{c}{2} = 20$	Simplify.	$20 - 13 \stackrel{?}{=} 7$
$2 \cdot \frac{c}{2} = 2 \cdot 20$	Multiplication Property of Equality	$7 = 7$ ✓
$c = 40$		

Example 2 **Solve $7y - 2y + 4 = 29$. Check your solution.**

$7y - 2y + 4 = 29$	Write the equation.	**CHECK:** $7y - 2y + 4 = 29$
$5y + 4 = 29$	Combine like terms.	$7(5) - 2(5) + 4 \stackrel{?}{=} 29$
$-4 = -4$	Subtraction Property of Equality	$35 - 10 + 4 \stackrel{?}{=} 29$
$5y = 25$	Simplify.	$25 + 4 \stackrel{?}{=} 29$
$\frac{5y}{5} = \frac{25}{5}$	Division Property of Equality	$29 = 29$ ✓
$y = 5$	Simplify. Check your solution.	

Exercises

Solve each equation. Check your solution.

1. $5t + 2 = 7$
2. $2x + 5 = 9$
3. $6u - 8 = 28$
4. $8m - 7 = 17$
5. $\frac{m}{7} - 9 = 5$
6. $\frac{k}{9} - 3 = -11$
7. $13 + \frac{a}{4} = -3$
8. $-3 + \frac{c}{2} = 12$
9. $7 - h = 209$
10. $-g + 18 = -32$
11. $15 - p = 3$
12. $-\frac{2}{5}c - 8 = 32$
13. $\frac{3}{8}q + 12 = 36$
14. $3 - \frac{3}{4}n = 9$
15. $\frac{7}{9}v + 2 = 23$
16. $7 + \frac{1}{8}l = -2$
17. $\frac{v}{-3} + 8 = 22$
18. $8x - 16 + 8x = 16$
19. $12a - 14a = 8$
20. $7c - 8 - 2c = 17$
21. $6 = -y + 42 - 2y$
22. $16 + 8r - 4r + 4 = 24$

NAME _____ DATE _____ PERIOD _____

4-5 Study Guide and Intervention *(continued)*

Solving Two-Step Equations

Solve Real-World Problems When solving two-step equations, always remember to add or subtract first and then multiply or divide to isolate the variable. This is the opposite of the order of operations.

Example **Nina read 50 pages of a 485-page book. Nina now plans to read 15 pages a day. The equation $50 + 15x = 485$ represents how many days it will take Nina to read the rest of the book. Write the steps that can be used to solve the equation.**

$50 + 15x = 485$	Write the equation.
$50 + 15x = 485$	
$-50 \quad = -50$	Subtraction Property of Equality
$15x = 435$	Simplify.
$\frac{15x}{15} = \frac{435}{15}$	Division Property of Equality
$x = 29$	Simplify.

To solve the equation, first subtract 50 and then divide by 15.

CHECK: $50 + 15x = 485$	Write the equation.
$50 + 15(29) \stackrel{?}{=} 485$	Substitute the solution for x.
$50 + 435 \stackrel{?}{=} 485$	Multiply.
$485 = 485$ ✓	Add.

Exercises

1. **FUNDRAISING** A high school band needs \$1,200 for a trip. So far they have raised \$430. They have 5 more fundraisers planned. The equation $\$430 + 5f = \$1{,}200$ represents how much money they must raise at each of the remaining fundraisers. List the series of steps you would take to solve the equation. Then give the solution.

2. **PRINTS** Haley bought a membership to an online photo-sharing site for \$12. After purchasing the membership, she wanted to buy several prints. Prints cost \$0.12 each. She has a total of \$18.00 to spend on both the membership and the prints. The equation $\$12 + \$0.12p = \$18$ represents how many prints Haley can purchase. List the series of steps you would take to solve the equation. Then give the solution.

3. **SAVINGS** Tim has \$85. He wants to save more money to buy a game system for \$390. He is able to save \$20 a week. The equation $\$85 + 20w = \$390$ represents how many weeks Tim must save. List the series of steps you would take to solve the equation. Then give the solution.

4. **CELL PHONES** A cell phone plan costs \$14.75 per month, plus \$0.18 cents per minute. Lisa has budgeted \$35 a month for her cell phone. The equation $\$14.75 + 0.18m = \$35$ represents how many minutes Lisa can use each month. List the series of steps you would take to solve the equation. Then give the solution.

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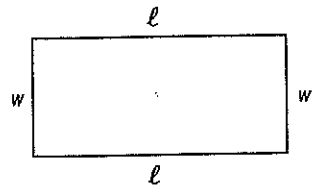
Study Guide and Intervention

Perimeter and Area

Perimeter Formulas are equations that show relationships among certain quantities. They usually contain two or more variables. You can use formulas to find the perimeter of a figure. **Perimeter** is the distance around a geometric figure.

Perimeter of a rectangle

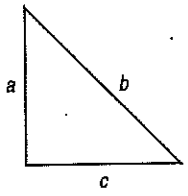


Words The perimeter of a rectangle is the sum of twice the length and twice the width.

Symbols $P = \ell + \ell + w + w$

$P = 2\ell + 2w$ or $2(\ell + w)$

Perimeter of a triangle

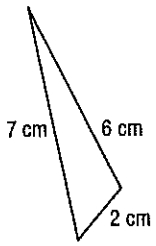


Words The perimeter of a triangle is the sum of the measure of all three sides.

Symbols $P = a + b + c$

Lesson 5-1

Example 1 **Find the perimeter of the triangle.**



$P = a + b + c$	Write the formula for perimeter.
$P = 7 + 6 + 2$	Replace a with 7, b with 6, and c with 2.
$P = 15$ cm	Simplify. The perimeter is 15 cm.

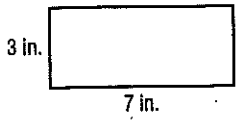
Example 2 **The perimeter of a rectangle is 26 inches. Its length is 7 inches. Find the width.**

$P = 2\ell + 2w$	Write the formula for perimeter.
$26 = 2 \cdot 7 + 2w$	Replace P with 26, and ℓ with 7.
$26 = 14 + 2w$	Simplify.
$26 - 14 = 14 - 14 + 2w$	Subtraction Property of Equality
$12 = 2w$	Simplify.
$\frac{12}{2} = \frac{2w}{2}$	Division Property of Equality
$6 = w$	Simplify. The width of the rectangle is 6 inches.

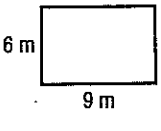
Exercises

Find the perimeter for each figure.

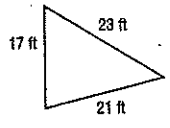
1.



2.



3.



4. Find the length of a rectangle if the width is 4.7 meters and the perimeter is 12.6 meters.

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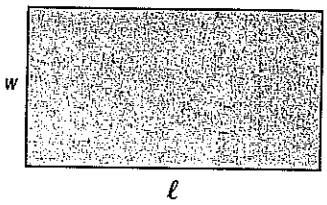
5-1 Study Guide and Intervention *(continued)*

Perimeter and Area

Area Formulas can also be used to calculate the area of a figure. **Area** is a measure of the surface enclosed by a figure and is always given in square units, u^2.

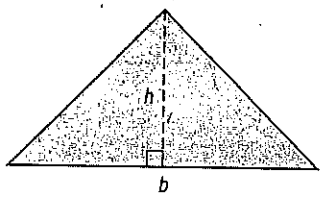
Area of a rectangle



Words The area of a rectangle is the product of the length and width.

Symbols $A = \ell w$

Area of a triangle



Words The area of a triangle is one-half the product of the base and height.

Symbols $A = \frac{1}{2}bh$

Example 1 **The base of a triangle is 14 feet and its height is 4.5 feet. Find its area.**

$A = \frac{1}{2}bh$ Write the formula for area.

$A = \frac{1}{2} \cdot 14 \cdot 4.5$ Replace b with 14 and h with 4.5.

$A = 31.5$ Simplify. The area is 31.5 square feet.

Example 2 **Find the length of a rectangle with an area of 54 square yards and a width of 8 yards.**

$A = \ell w$ Write the formula for area.

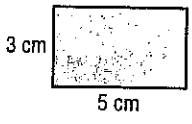
$54 = 8\ell$ Replace A with 54 and w with 8.

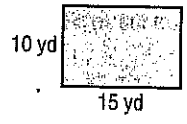
$\frac{54}{8} = \frac{8\ell}{8}$ Division Property of Equality

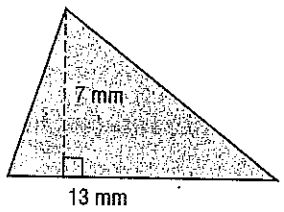
$6.75 = \ell$ Simplify. The length is 6.75 yards.

Exercises

Find the area for each figure.

1. 

2. 

3. 

4. Find the height of a triangle if the area is 48 square millimeters and the base is 24 millimeters.

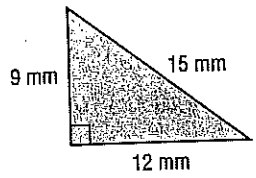
NAME _____ DATE _____ PERIOD _____

5-1 Skills Practice

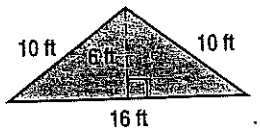
Perimeter and Area

Find the perimeter and area for each figure.

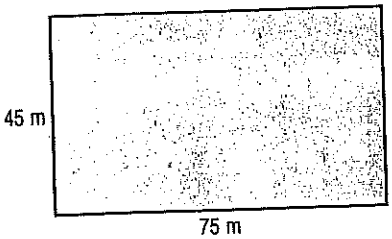
1.



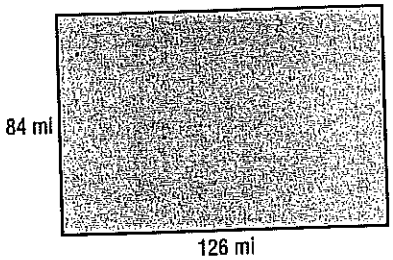
2.



3.



4.



5. a right triangle with legs of 7 inches and 24 inches and a hypotenuse of 25 inches

6. a rectangle that is 21 inches long and 13 inches wide

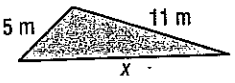
7. a square that is 25 centimeters on each side

Find the missing dimension for each figure.

8.

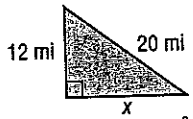
Perimeter = 208 in.

9.



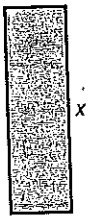
Perimeter = 30 m

10.



Area = 96 m²

11.



Area = 216 ft²

12. The perimeter of a rectangle is 100 centimeters. Its width is 9 centimeters. Find its length.

13. The area of a rectangle is 319 square kilometers. Its width is 11 kilometers. Find its length.

14. The perimeter of a triangle is $37\frac{1}{2}$ yards. Two of the sides each measure $12\frac{1}{8}$ yards. Find the third side.

Lesson 5-1

NAME _____ DATE _____ PERIOD _____

5-2 Study Guide and Intervention *(continued)*

Solving Equations with Variables on Each Side

Write Equations with Variables On Each Side You can write equations with variables on each side to solve word problems.

Example **SHOPPING** **Maya bought a pair of boots for $32 and then bought 3 T-shirts. Paul bought a cap for $12 and then bought 5 T-shirts. If all the T-shirts cost the same amount, and Maya and Paul spent the same amount in all, write and solve an equation to find the cost of one T-shirt.**

Words	cost of boots	+	number of T-shirts	×	cost per T-shirt	=	cost of cap	+	number of T-shirts	×	cost per T-shirt
Variable	Let t = the cost of one T-shirt										
Equation	32	+	$3t$			=	12	+	$5t$		

$32 + 3t = 12 + 5t$	Write the equation.
$32 + 3t - 3t = 12 + 5t - 3t$	Subtraction Property of Equality
$32 = 12 + 2t$	Simplify.
$32 - 12 = 12 - 12 + 2t$	Subtraction Property of Equality
$20 = 2t$	Simplify.
$10 = t$	Mentally divide each side by 2.

The cost for one T-shirt is $10.

Exercises

1. **PHONES** Acme Phone Company charges $21 a month plus $0.05 a minute. Belltone Phones charges $15 a month plus $0.11 a minute. Write and solve an equation to determine how many minutes a month you must use for the costs of using either company to be equal.

2. **PARTIES** Mrs. Lin is planning her daughter's birthday party. At Parties R Us, the fee is $80 plus $10 per child. At the Birthday Palace, the fee is $150 plus $5 per child. Write and solve an equation to determine how many children must be invited for the costs to be equal.

3. **POOLS** A town pool has two individual membership rates. You can pay a $75 membership fee and then $2 each time you use the pool or you can pay a $15 membership fee and $5 each time you use the pool. Write and solve an equation to determine how many times you must visit the pool for the costs to be equal.

4. **TAXI** Speedy Cab has an initial charge of $2.50 plus $3.50 for each additional mile. Friendly Cab has an initial charge of $5.50 plus an additional $2.00 per mile. Write and solve an equation to determine how many miles you must go for the costs to be equal.

Lesson 5-2

NAME _____ DATE _____ PERIOD _____

5-5 Skills Practice

Solving Equations with Variables on Each Side

Solve each equation. Check your solution.

1. $3x + 2 = 5x$
2. $n - 12 = 3n$
3. $2 - 3b = 7b + 12$
4. $4d - 11 = 2d + 7$
5. $2f + 3 = 11f - 24$
6. $8y + 11 = 2y + 29$
7. $5a = 45 + 2a$
8. $17 - 3c = 4c + 3$
9. $2a - 3 = 9a - 10$
10. $5b = 21 + 4b$
11. $9y - 27 = -2y + 6$
12. $2n - 5 = 7n$
13. $-s + 3 = 5s + 21$
14. $7 - 4c = 3c + 35$
15. $30 - 2n = 4n$
16. $29 + 7d = 5d + 15$
17. $16k - 23 = 6k - 13$
18. $w - 20 = 6w$
19. $33g + 28 = 25g - 12$
20. $6h - 34 = -6h + 14$
21. $3t + 17 = t - 3$
22. $11j = 6j - 15$
23. $c - 2 = 3c + 14$
24. $28x - 7 = 26x + 5$
25. $5m - 6 = 8m + 9$
26. $-4p - 7 = 5p + 11$
27. $-10 + 3f = 5f + 6$
28. $4f + 6 = 8f - 14$
29. $-7n - 16 = 4n + 17$
30. $5d = 9d - 18$

Define a variable and write an equation to find each number. Then solve.

31. Three times a number equals 40 more than five times the number. What is the number?

32. A number equals four less than three times the number. What is the number?

33. Eight times a number equals 24 more than two times the number. What is the number?

NAME _____ DATE _____ PERIOD _____

5-3 Study Guide and Intervention *(continued)*

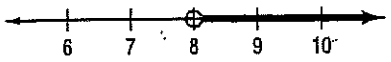
Inequalities

Graph Inequalities Inequalities can be graphed on a number line. This helps you see which values make the inequality true. You can also write inequalities for a graph.

An *open dot* indicates that the number marked *does not* make the sentence true.
A *closed dot* indicates that the number marked *does* make the sentence true.
The direction of the line indicates whether numbers *greater than* or *less than* the number marked make the sentence true.

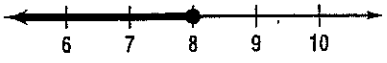
Example 1 **Graph each inequality on a number line.**

a. $x > 8$



The **open dot** means 8 *does not* make the sentence true. The line means that numbers greater than 8 make the sentence true.

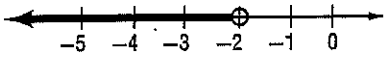
b. $x \leq 8$



The **closed dot** means 8 *does* make the sentence true. The line means that numbers less than 8 make the sentence true.

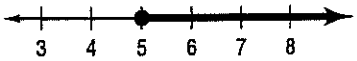
Example 2 **Write an inequality for each graph.**

a.



The open dot means −2 is not included in the graph. The arrow points left, so the graph includes all numbers less than −2.
The inequality is $x < -2$.

b.

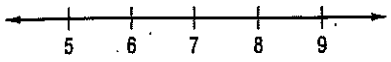


The closed dot means 5 is included in the graph. The arrow points right, so the graph includes all numbers greater than 5.
The inequality is $x \geq 5$.

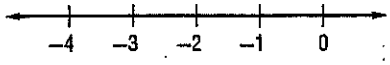
Exercises

Graph each inequality on a number line.

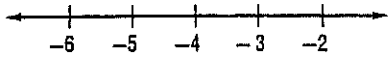
1. $x > 7$



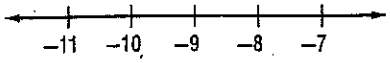
2. $a \leq -2$



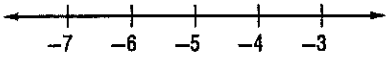
3. $d < -4$



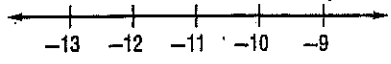
4. $w > -9$



5. $t \geq -5$

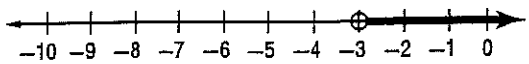


6. $n < -11$



Write the inequality for each graph.

7.



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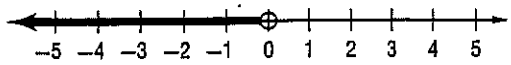

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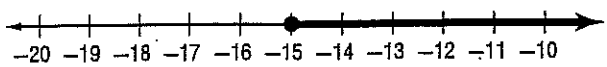
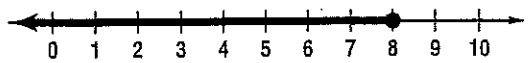


10.



NAME _____ DATE _____ PERIOD _____

Skills Practice

Inequalities

Write an inequality for each sentence.

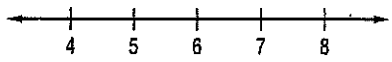
1. More than 100,000 fans attended the opening football game at The Ohio State University.
2. Her earnings at \$16 per hour were no more than \$96.
3. A savings account decreased by \$50 is now less than \$740.
4. A number increased by 7 is at least 45.

For the given value, state whether each inequality is *true* or *false*.

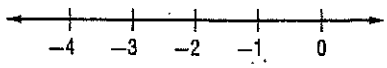
5. $\frac{18}{c} < 9, c = 2$
6. $\frac{x}{5} \geq 3, x = 5$
7. $6k \geq 42, k = 7$
8. $10 - x < 3, x = 7$
9. $11 + n < 32, n = 4$
10. $9 + c > 19, c = 10$

Graph each inequality on a number line.

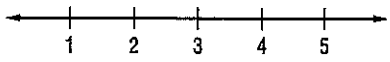
11. $a < 6$



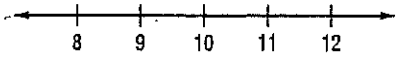
12. $t \geq -2$



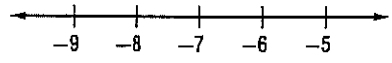
13. $d \leq 3$



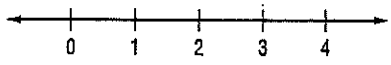
14. $b \geq 10$



15. $x \geq -7$

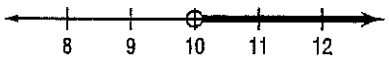


16. $x > 2$



Write the inequality for each graph.

17.



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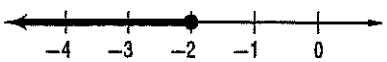

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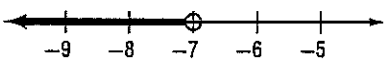

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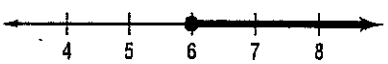

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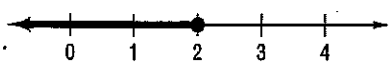

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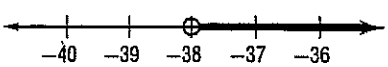


NAME _____ DATE _____ PERIOD _____

5-3 Practice

Inequalities

Write an inequality for each sentence.

1. More than 3400 people attended the flea market.

2. Her earnings at \$11 per hour were no more than \$121.

3. The 10-km race time of 84 minutes was at least twice as long as the winner's time.

4. A savings account increased by \$70 is now more than \$400.

For the given value, state whether each inequality is *true* or *false*.

5. $9 - x > 3, x = 6.5$

6. $9.5 + n < 19, n = 10$

7. $3k < 27\frac{1}{2}, k = 8$

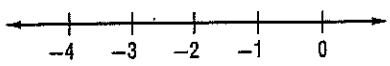
8. $21 \leq 4c, c = 5.2$

9. $\frac{x}{4} \leq 8, x = 32$

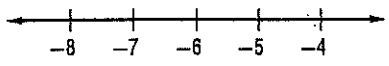
10. $\frac{9}{c} > 2, c = 3\frac{1}{2}$

Graph each inequality on a number line.

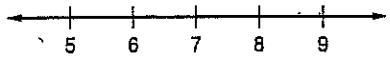
11. $a < -2$



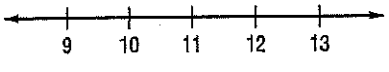
12. $t > -6$



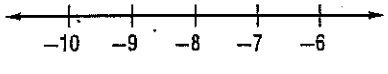
13. $d \geq 7$



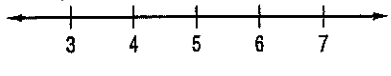
14. $b \geq 11$



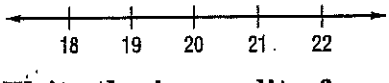
15. $x \leq -8$



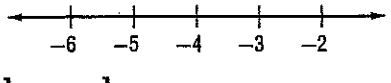
16. $w > 5$



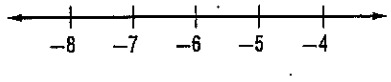
17. $n < 20$



18. $b \leq -4$

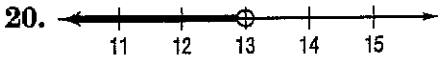


19. $a \geq -6$



Write the inequality for each graph.

20.



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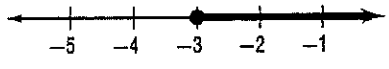

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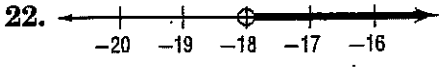

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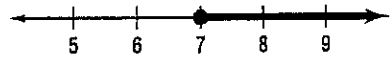

24. **HIPPOS** The average time a human being can hold their breath underwater is 1 minute. A hippo can hold its breath underwater for at least 5 times as long as a human. Write an inequality that represents how long a hippo can hold its breath underwater.

25. **CHARITY** In the first hour of a charity auction, \$4800 was raised. This was at most \$1200 more than was raised in the second hour of the auction. Write an inequality that represents the amount raised in the second hour.